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the lectures pdfs are available at:

https://www.physics.umd.edu/rgroups/amo/orozco/results/2022/Results22.htm

Correlations in Optics and Quantum Optics; A series of lectures about correlations and coherence. November 2022

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Lesson 6

Tentative list of topics to cover:

- From statistics and linear algebra to power spectral densities
- Historical perspectives and examples in many areas of physics
- Correlation functions in classical optics (field-field; intensity-intensity; field-intensity) part iii
- Optical Cavity QED
- Correlation functions in quantum examples
- Correlations of the field and intensity
- Correlations and conditional dynamics for control
- From Cavity QED to waveguide QED.

Cavity QED in the optical regime

Quantum electrodynamics for pedestrians. There is no need to renormalize. There is only one mode of the electromagnetic field.

ATOM(S) + CAVITY

Perturbative regime: coupling < dissipation.
Decay rate increase or decrease (cavity less than $\lambda/2$), changes in energy levels.
Non-perturbative regime: coupling > dissipation. Splitting of the levels by the coupling (Vacuum Rabi).

Beer-Lambert law for intensity attenuation

$\frac{dI}{dz} = -\alpha I$ if α is resonant and independent

of I, α_0 (does not saturate)

$I = I_0 \exp(-\alpha_0 l)$

where $\alpha_0 = \sigma_0 \rho$

and $\rho = N / V$ the density of absorbers in a length l

Rate of decay
(Fermi's golden rule)

$$\gamma_{rad} \approx \frac{2\pi}{\hbar} \rho(k) \langle H_{int} \rangle^2$$

Phase space density Interaction

Rate of decay free space (Fermi's golden rule)

$$\gamma_0 = \frac{\omega_0^3 d^2}{\pi \varepsilon_0 \hbar c^3}$$

Where *d* is the dipole moment

Saturation intensity:
One photon every two lifetimes over the cross section of the atom (resonant)

$$I_s = \frac{\hbar\omega_0}{2\tau_0\sigma_0} = \frac{\pi}{3}\frac{\gamma_0\hbar\omega_0}{\lambda_0^2}$$

If $I=I_0$ the rate of stimulated emission is equal to the rate of spontaneous emission (Rabi Frequency Ω) and the population on the excited state 1/4.

$$\Omega = \frac{\vec{d} \bullet \vec{E}}{\hbar} = \gamma \sqrt{I/I_s}$$

$$\text{Excited Population} = \frac{1}{2} \frac{I/I_s}{1 + I/I_s}$$

Energy due to the interaction between a dipole and an electric field.

$$H = \vec{d} \cdot \vec{E}$$

The dipole matrix element between two states is fixed by the properties of the states (radial part) and the Clebsh-Gordan coefficients from the angular part of the integral. It is a few times a_0 (Bohr radius) times the electron charge e between the S ground and P first excited state in alkali atoms.

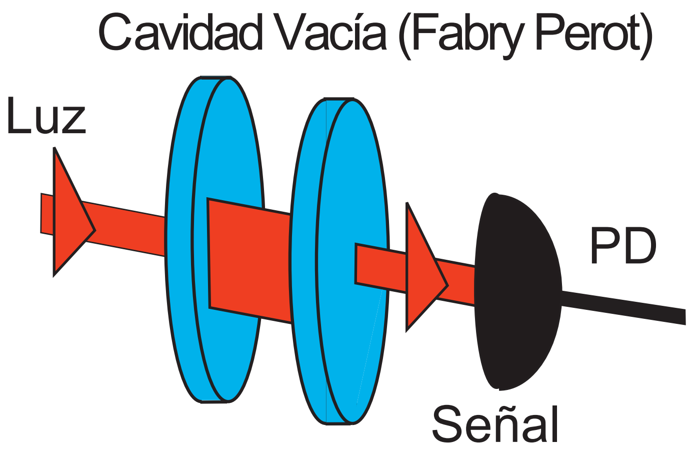
$$\vec{d} = e\left\langle 5S_{1/2}\left|\vec{r}\right|5P_{3/2}\right\rangle$$

The dipole coupling between the atom and the cavity:

$$g = \frac{d \cdot E_v}{\hbar}$$

The field of a photon in a cavity with volume V_{eff} is:

$$E_v = \sqrt{\frac{\hbar\omega}{2\varepsilon_0 V_{eff}}}$$


Cavidad Vacía (Fabry Perot)
Luz
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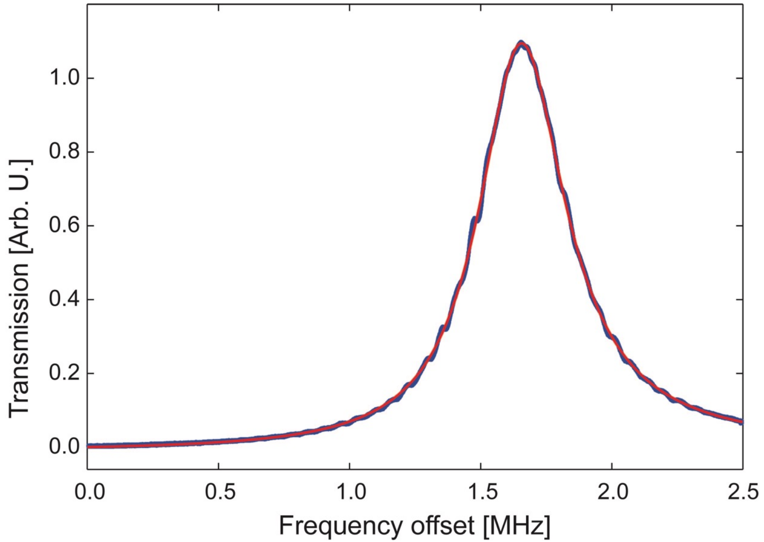
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0.8
0.6
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0.0
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0.5
1.0
1.5
2.0
2.5
Frequency offset [MHz]

Density matrix

em drive

atoms-em mode coupling

$$\dot{\rho} = \mathcal{E}[\hat{a}^{\dagger} - \hat{a}, \rho] + g[\hat{a}^{\dagger}\hat{J}_{-} - \hat{a}\hat{J}_{+}, \rho]$$

$$+ \kappa(2\hat{a}\rho\hat{a}^{\dagger} - \hat{a}^{\dagger}\hat{a}\rho - \rho\hat{a}^{\dagger}\ddot{a})$$

Cavity decay

$$+ (\gamma/2)\sum_{j=1}^{N}(2\hat{\sigma}^{j}_{-}\rho\hat{\sigma}^{j}_{+} - \hat{\sigma}^{j}_{+}\hat{\sigma}^{j}_{-}\rho - \rho\hat{\sigma}^{j}_{+}\hat{\sigma}^{j}_{-}),$$

Atomic decay

Decorrelated equations:

Radiation field:

$$\frac{\partial}{\partial t}\langle \hat{a} \rangle = -\kappa(1+i\theta)\langle \hat{a} \rangle + \sum_{j=1}^{N} g_j \langle \hat{\sigma}_j^- \rangle + \mathcal{E},$$

Atomic polarization:

$$\frac{\partial}{\partial t}\langle \hat{\sigma}_j^- \rangle = -\gamma_\perp(1+i\Delta)\langle \hat{\sigma}_j^- \rangle + g_j \langle \hat{a} \rangle \langle \hat{\sigma}_j^z \rangle,$$

Atomic inversion:

$$\frac{\partial}{\partial t}\langle \hat{\sigma}_j^z \rangle = -\gamma_\parallel \left(\langle \hat{\sigma}_j^z \rangle + 1 \right) - 2g_j \left(\langle \hat{a} \rangle \langle \hat{\sigma}_j^+ \rangle + \langle \hat{a}^\dagger \rangle \langle \hat{\sigma}_j^- \rangle \right).$$

The cavity and atomic detunings θ and Δ are defined as

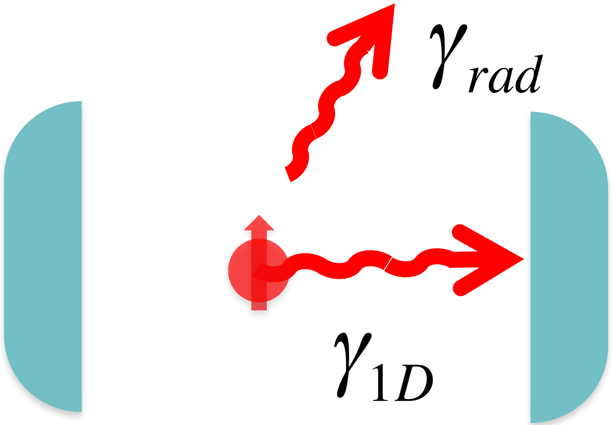
$$\theta = \frac{\omega_c - \omega_l}{\kappa} \quad \text{and} \quad \Delta = \frac{\omega_a - \omega_l}{\gamma_\perp}.$$

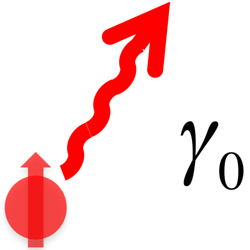
A first introduction to the Cooperativity

- Atomic decay rate γ
- Cavity decay rate κ
- Atom-cavity coupling rate g

$$C_1 = \frac{g^2}{\kappa\gamma} \qquad C = NC_1$$

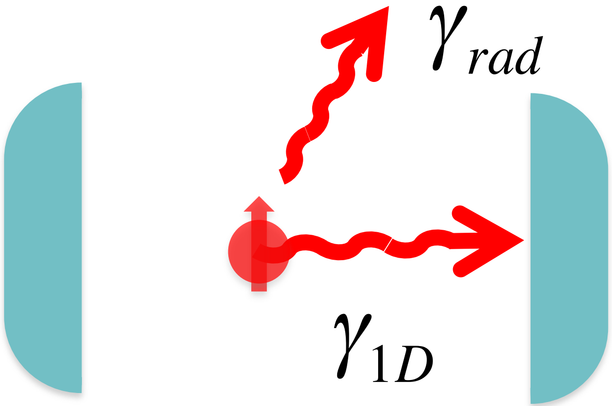
Coupling Enhancement

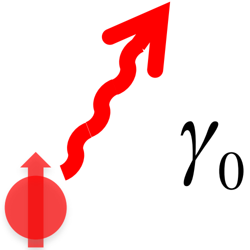




$$\alpha = \frac{\gamma_{1D}}{\gamma_0}$$

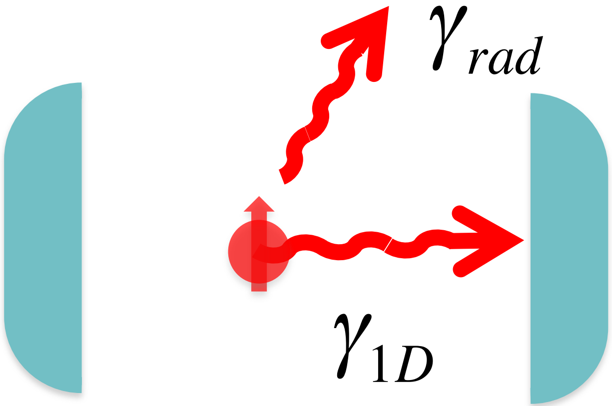
Coupling Efficiency

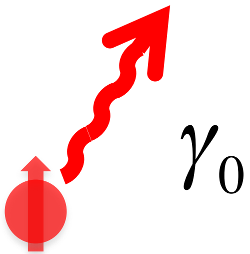




$$\beta = \frac{\gamma_{1D}}{\gamma_{Tot}} \quad ; \quad \gamma_{Tot} = \gamma_{1D} + \gamma_{rad}$$

Purcell Factor

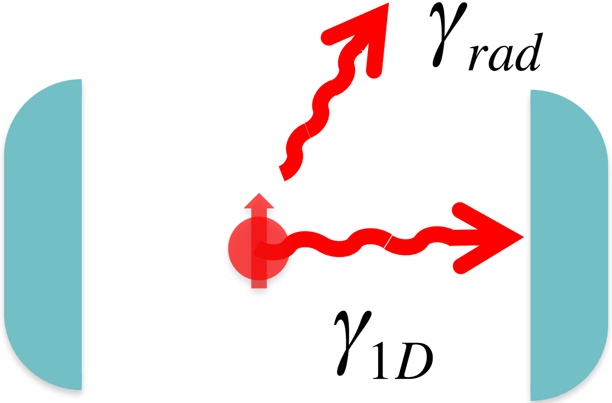


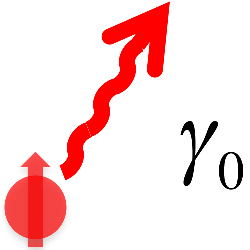


$$F_P = \frac{\gamma_{tot}}{\gamma_0} = \frac{\alpha}{\beta}$$

$$\gamma_{Tot} = \gamma_{1D} + \gamma_{rad}$$

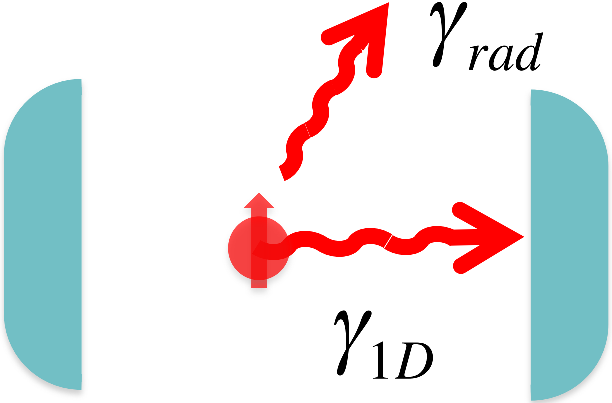
Cooperativity

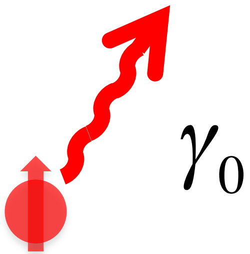




$$C_1 = \frac{\beta}{(1-\beta)} = \frac{\gamma_{1D}}{\gamma_{rad}}$$

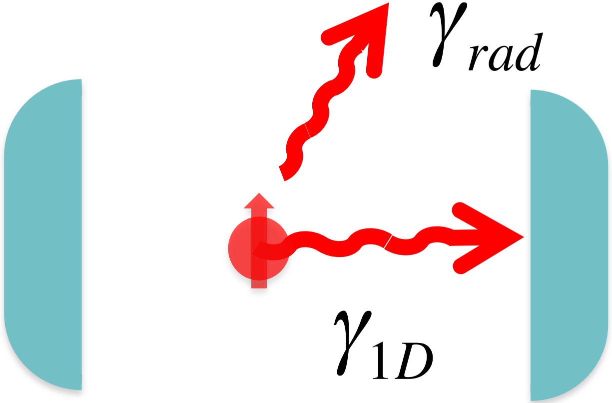
Cooperativity

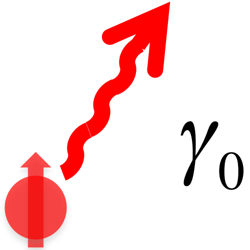




$$C_1 = \frac{g^2}{\kappa \gamma_0}$$

Cooperativity





$$C_1 = \frac{\sigma_0}{Area_{\mathrm{mode}}} \frac{1}{T}$$

Dimensionless input and output field normalized to the saturation intensity I_s and the transmission coefficient T of the output mirror

$$y = \frac{E_I}{\sqrt{I_s}}; \text{ and } x = \frac{E_T}{\sqrt{TI_s}}:$$

Low intensity $x<<1$: with noinversion, resonant $\Delta=0$ and $\Theta=0$ weakly driven.

Two coupled oscillators

$$\dot{x} = \kappa(-x + 2Cp + y)$$

$$\dot{p} = \gamma(-p - x)$$

Two coupled oscillators

Steady state

$$y = x - 2Cp$$

$$p = -x$$

$$y = x(1 + 2C)$$

$$\kappa >> \gamma \quad \dot{p} = -\gamma(1 + 2C)p - \gamma y$$

$$\gamma >> \kappa \quad \dot{x} = -\kappa(1 + 2C)x + \kappa y$$

Enhanced emission

Steady state with detuning and at all intensities:

$$y = x\left(1 + \frac{2C}{1+\Delta^2+|x|^2}\right) + ix\left(\theta - \frac{2C\Delta}{1+\Delta^2+|x|^2}\right)$$

Dispersive limit when Θ=0 and Δ >> 1 :

$$y = -ix\frac{2C\Delta}{1+\Delta^2+|x|^2}$$

Two coupled oscillators

$$\frac{x}{y} = \frac{A}{i\Omega - \Omega_1} + \frac{B}{i\Omega - \Omega_2} ,$$

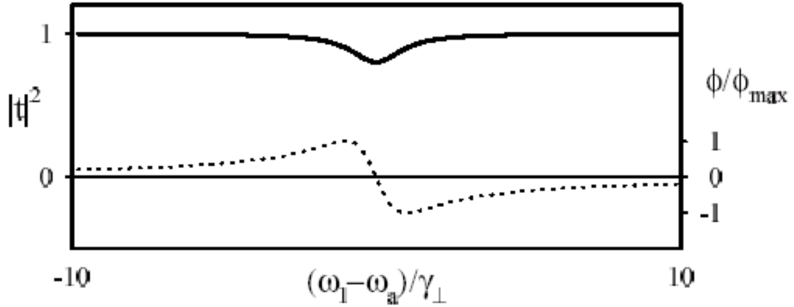
$$A = \kappa \frac{\gamma_\perp + \Omega_1}{\Omega_1 - \Omega_2} ,$$

$$B = \kappa \frac{\gamma_\perp + \Omega_2}{\Omega_2 - \Omega_1} ,$$

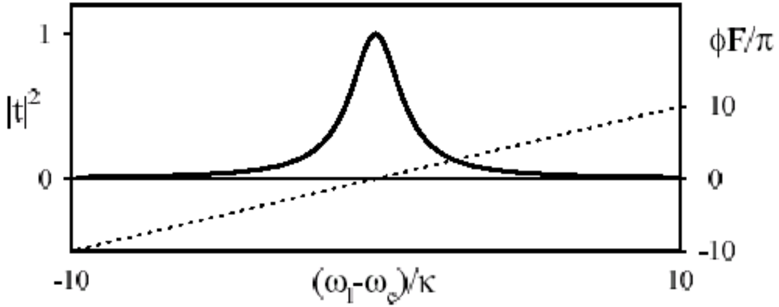
$$\Omega_{1,2} = -\frac{\kappa + \gamma_\perp}{2} \pm i \sqrt{-\left(\frac{\kappa - \gamma_\perp}{2}\right)^2 + \frac{\Omega_{V.R.}^2}{1 + \gamma_\perp^2 |x|^2 / (\gamma_\perp^2 + \Omega^2)}} .$$

Cavity mode and atomic polarization

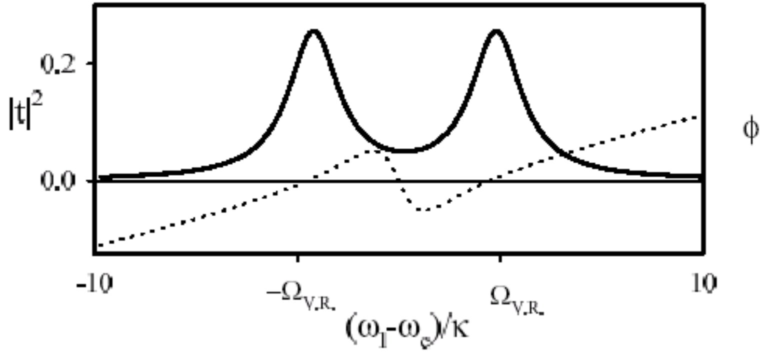
Atomic absorption



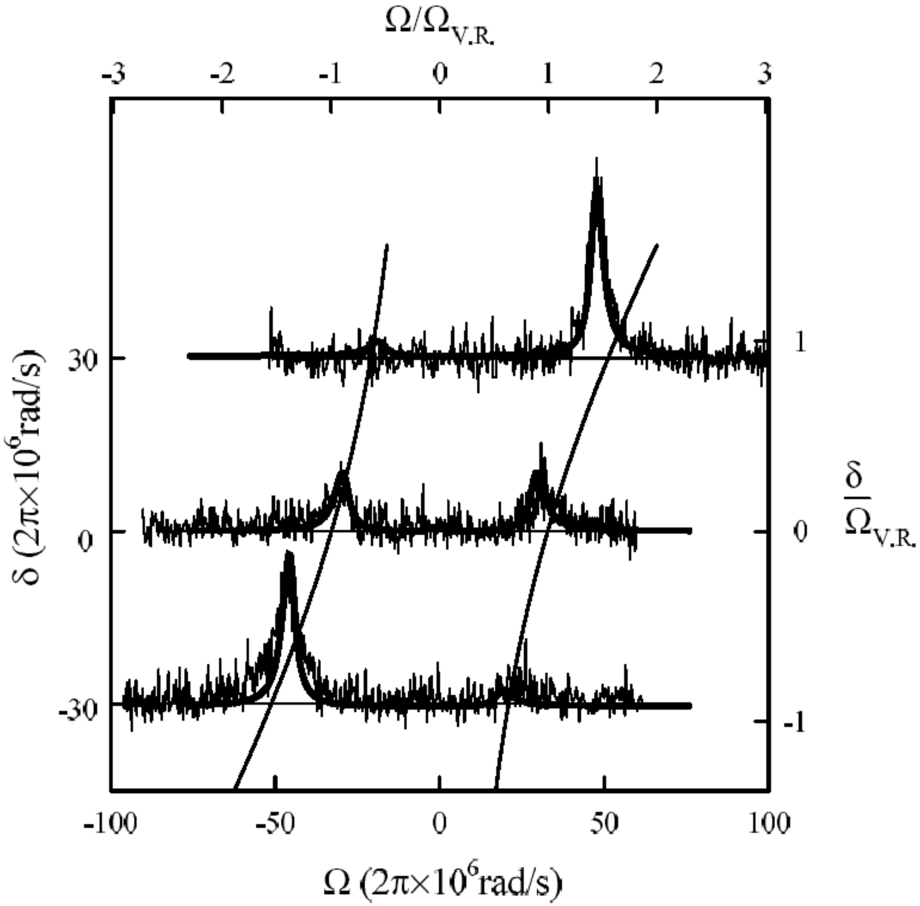
Fabry Perot



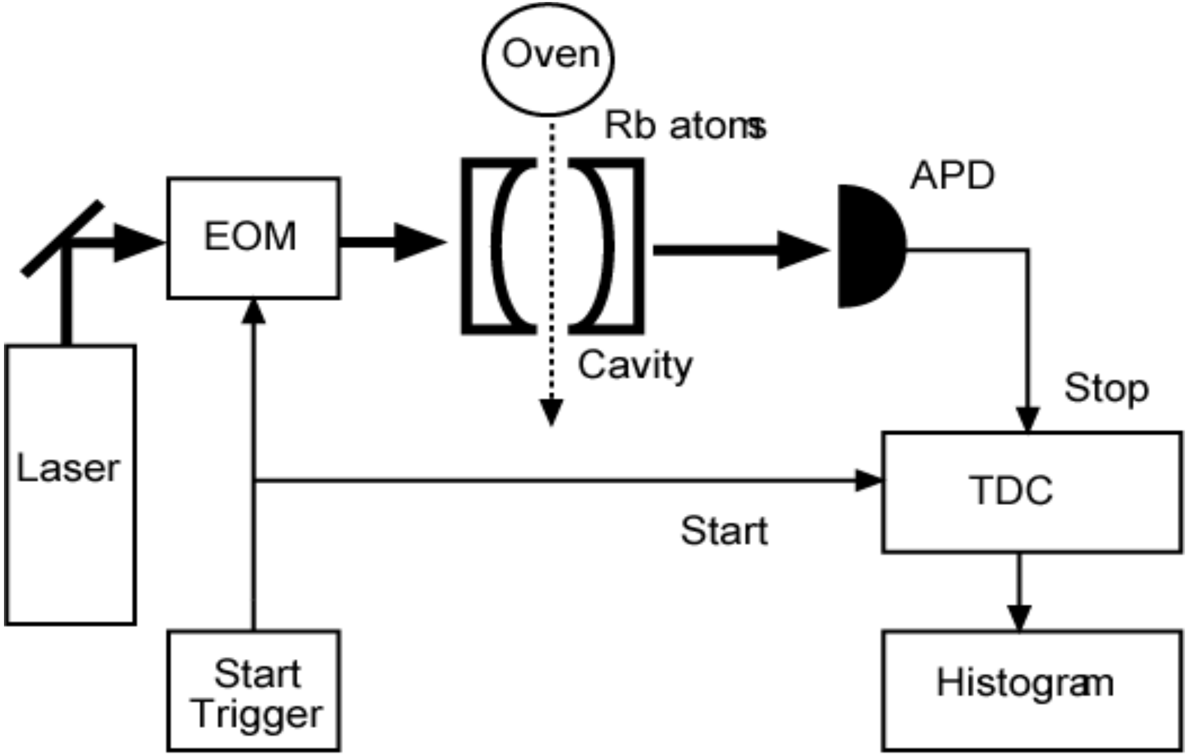
Coupled modes



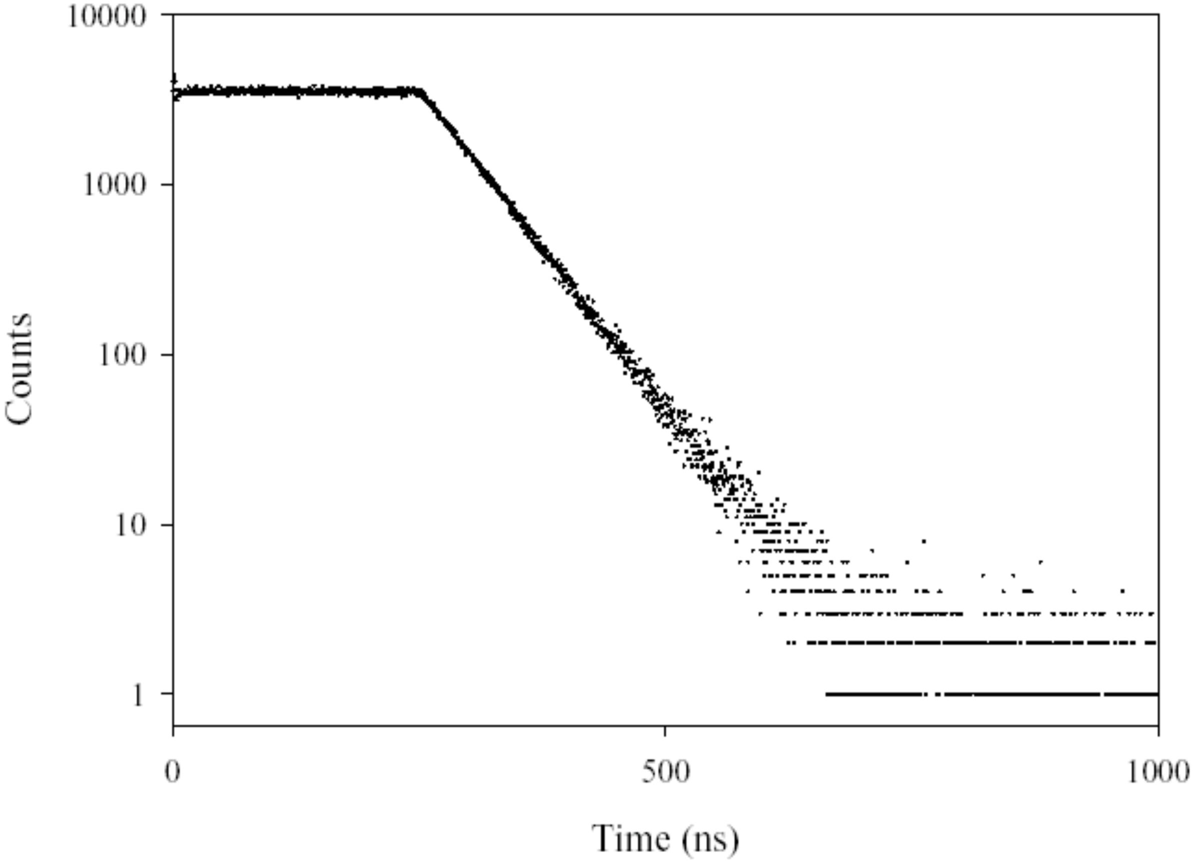
Transmission spectrum at low intensity for varying atomic detunings.



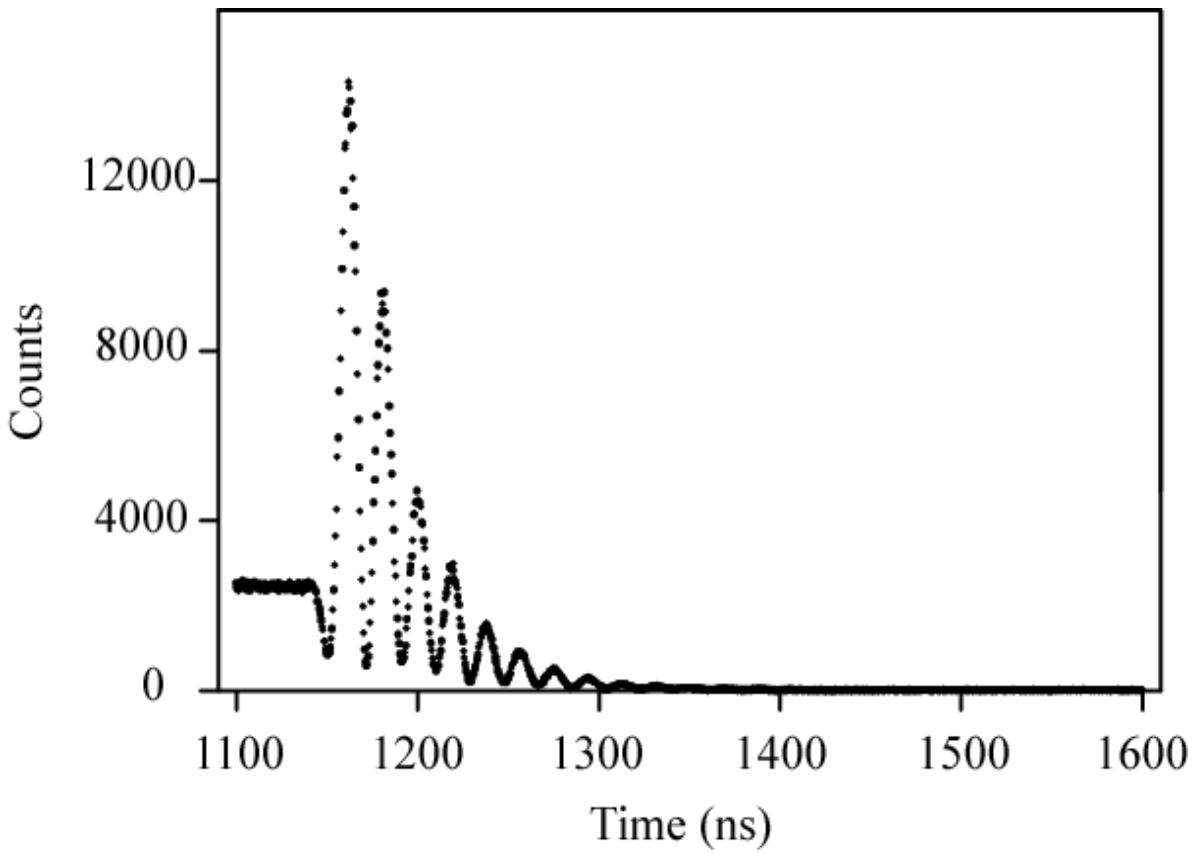
Study the system dynamics by providing a step function.



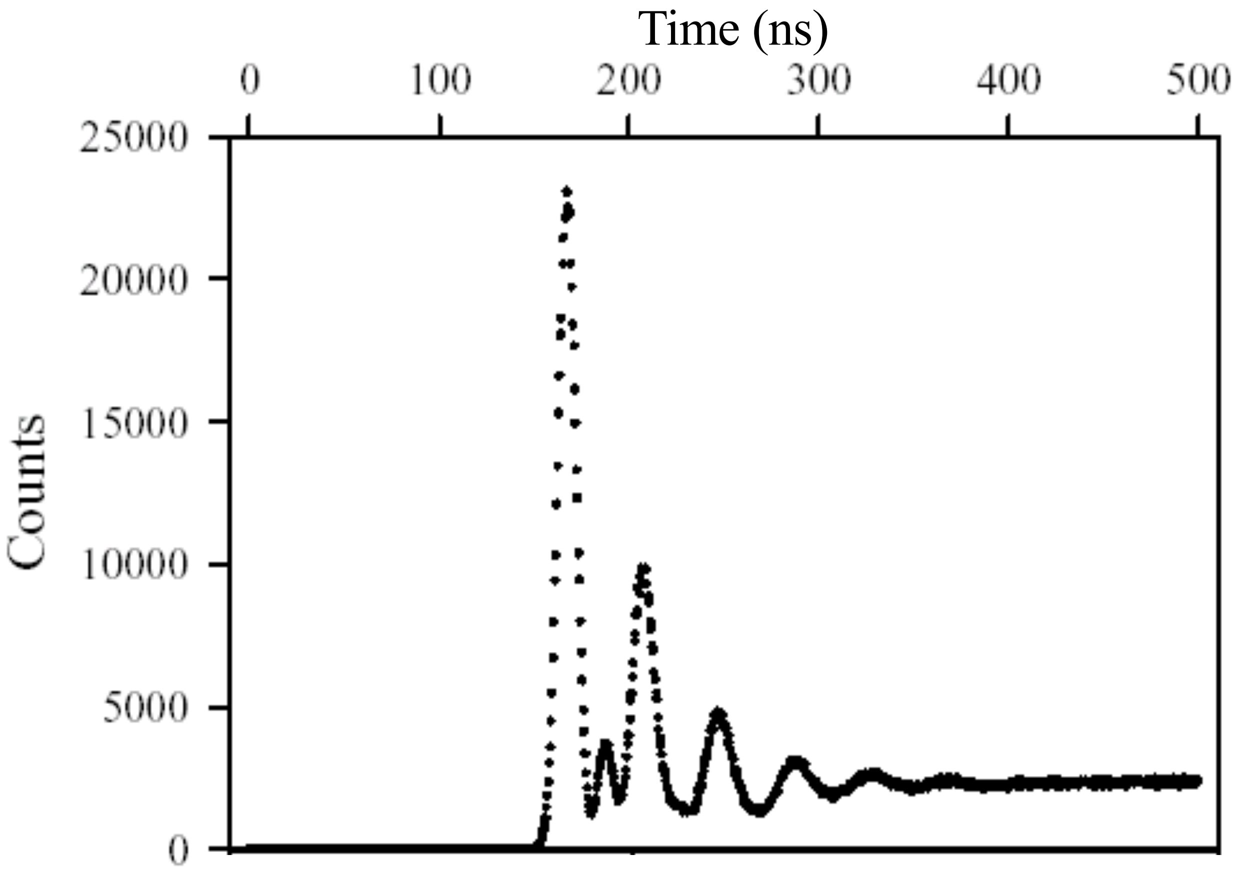
Decay of the empty cavity



Response to step down excitation



Response to step up excitation



Hamiltonian for *N* atoms

$$\hat{H} = \hat{H}_1 + \hat{H}_1 + \hat{H}_2 + \hat{H}_3 + \hat{H}_4 + \hat{H}_5 \ ,$$

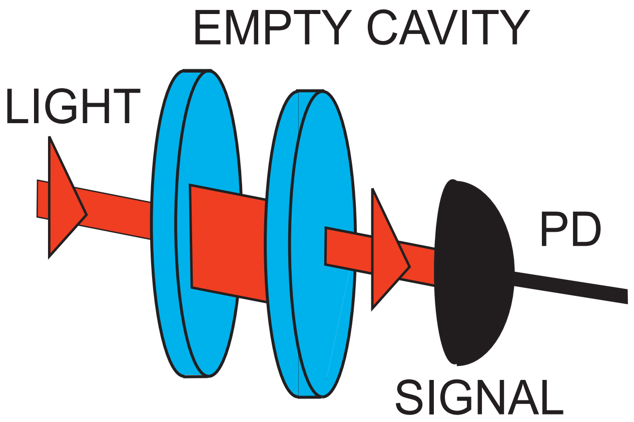
$$\hat{H}_1 = \hbar\omega_c \hat{a}^\dagger \hat{a} + \frac{1}{2}\hbar\omega_a \sum_{j=1}^{N} \hat{\sigma}_j^z \ ,$$ Free atoms free field

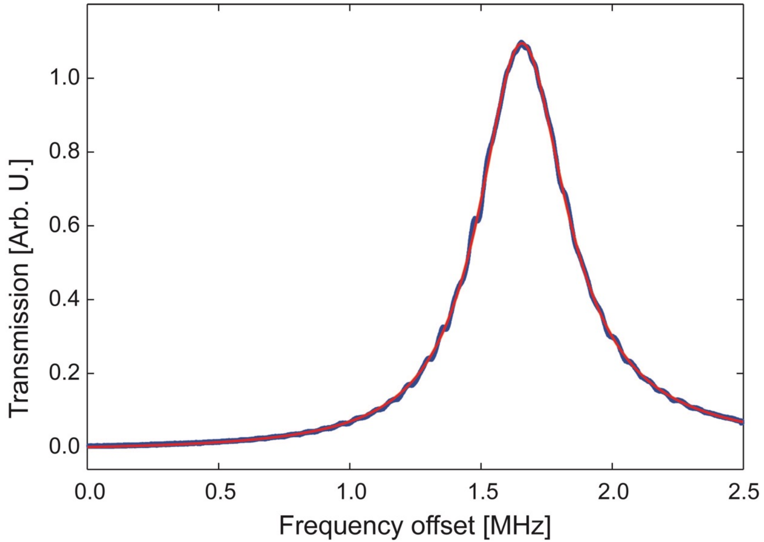
J.C $$\hat{H}_2 = i\hbar \sum_{j=1}^{N} g_j \left(\hat{a}^\dagger \hat{\sigma}_j^- e^{-i\vec{k}\cdot\vec{r}_j} - \hat{a}\hat{\sigma}_j^+ e^{i\vec{k}\cdot\vec{r}_j} \right)$$ Interaction

$$\hat{H}_3 = \sum_{j=1}^{N} \left(\hat{\Gamma}_A \hat{\sigma}_j^+ + \hat{\Gamma}_A^\dagger \hat{\sigma}_j^- \right) \ ,$$ Atomic decay

$$\hat{H}_4 = \hat{\Gamma}_F \hat{a}^\dagger + \hat{\Gamma}_F^\dagger \hat{a} \ ,$$ Cavity decay

$$\hat{H}_5 = i\hbar \left(\hat{a}^\dagger \mathcal{E} e^{-i\omega_l t} - \hat{a}\mathcal{E}^* e^{i\omega_l t} \right) \ .$$ Drive


EMPTY CAVITY
LIGHT
PD
SIGNAL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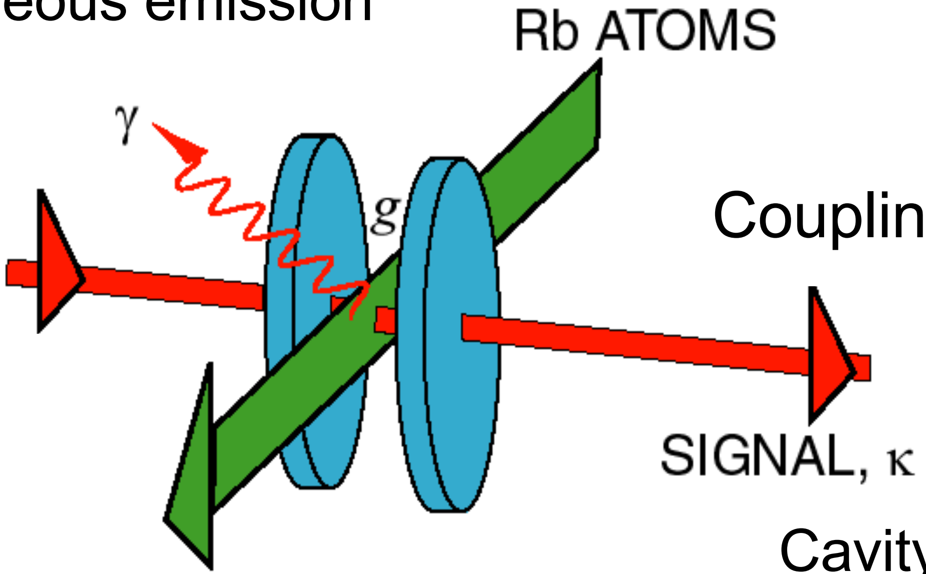
Frequency offset [MHz]

Spontaneous emission



Coupling

Cavity decay

Cooperativity for one atom: C_1

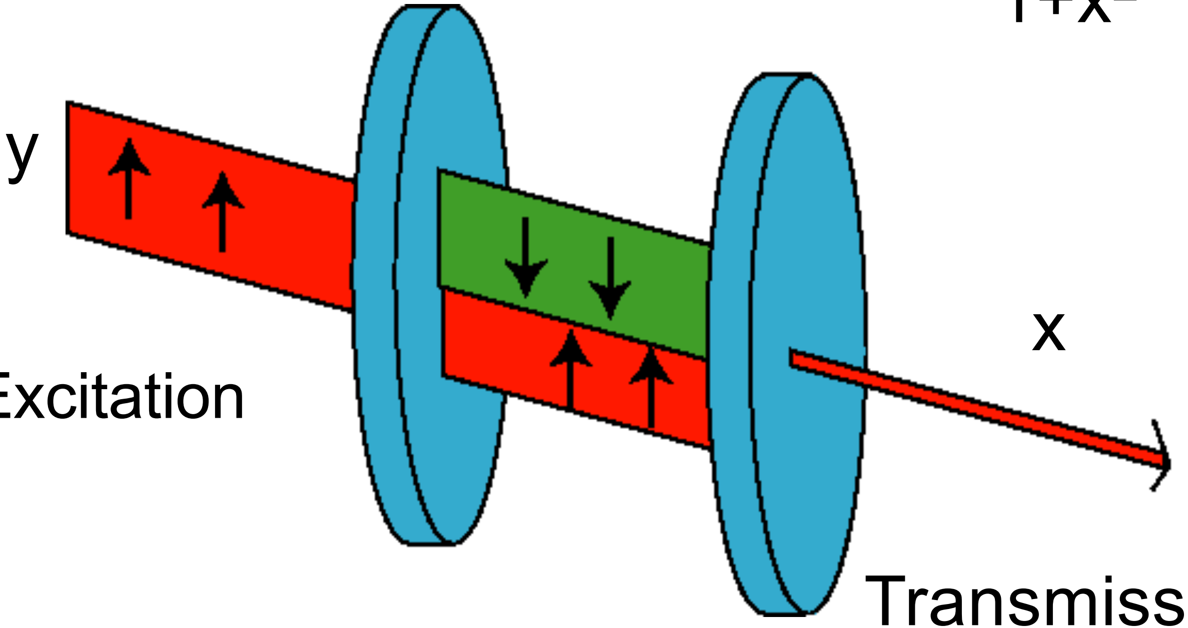
$$C_1 = \frac{g^2}{\kappa\gamma} \qquad C = C_1 N$$

Cooperativity for N atoms: C

$$g \approx \kappa \approx \gamma$$

Steady State

Atomic polarization: $\dfrac{-2Cx}{1+x^2}$



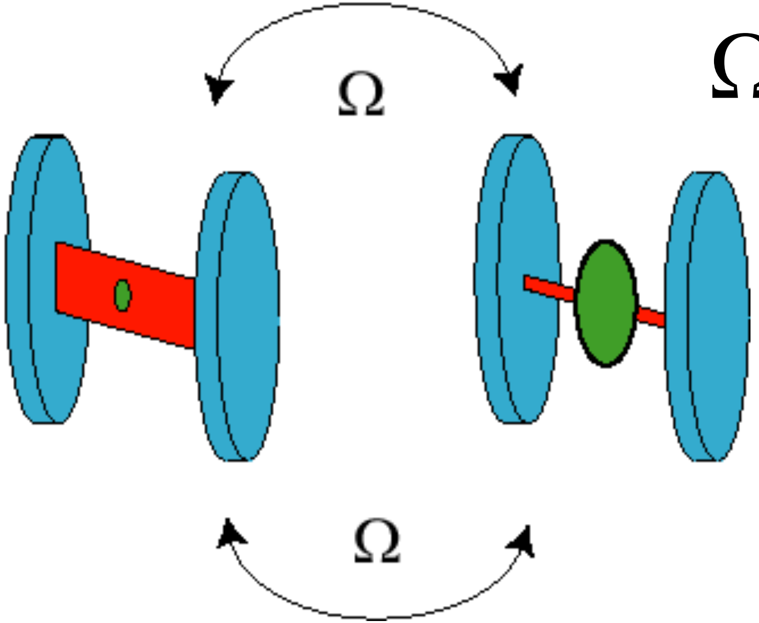
Excitation

Transmission

$x/y = 1/(1+2C)$

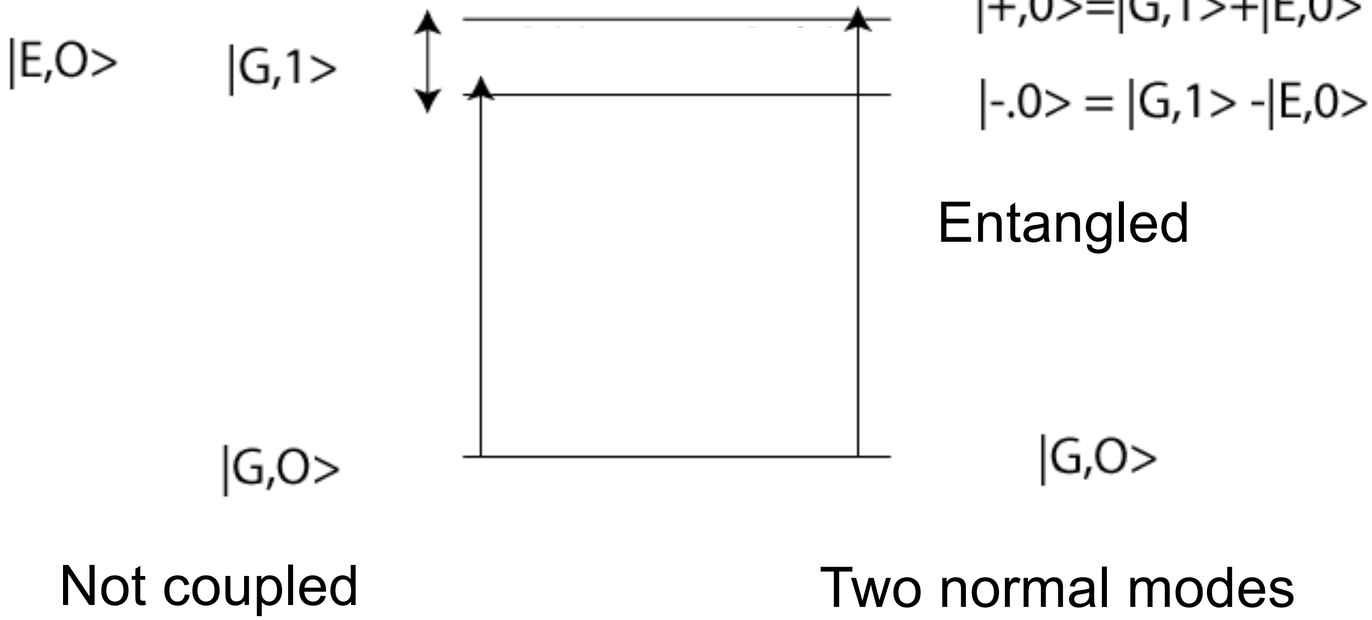
Jaynes Cummings Dynamics
Rabi Oscillations

Exchange of excitation for *N* atoms:

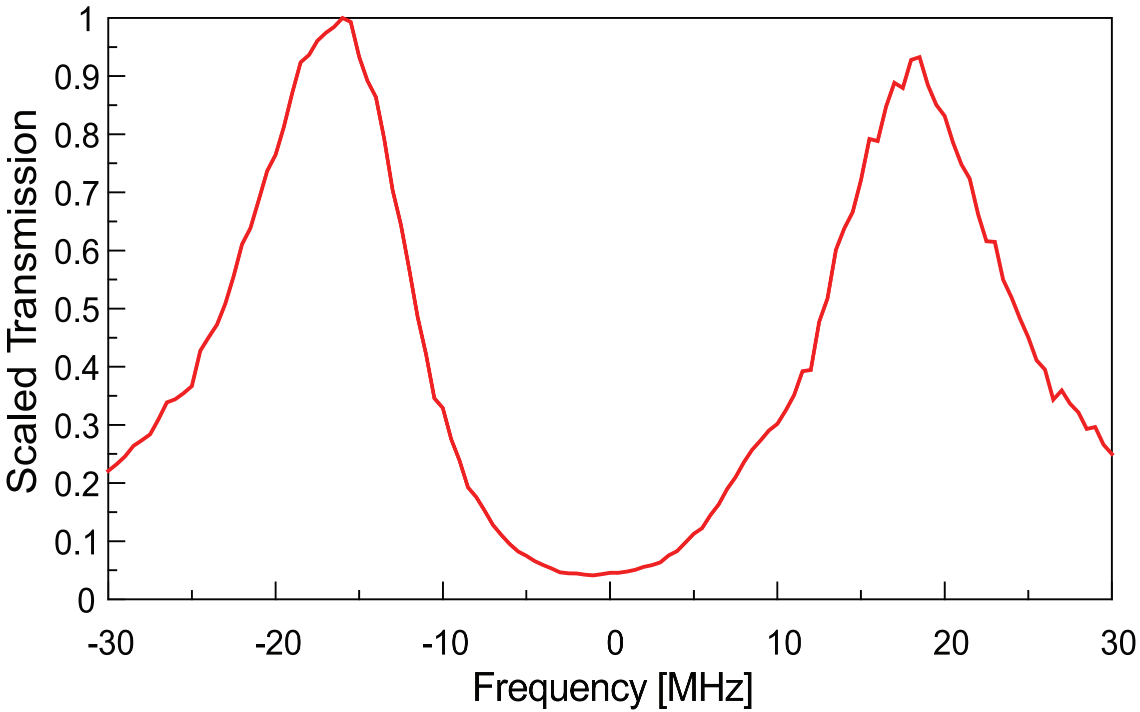


$$\Omega \approx g\sqrt{N}$$

2g Vacuum Rabi Splitting



Transmission doublet different from the Fabry Perot resonance



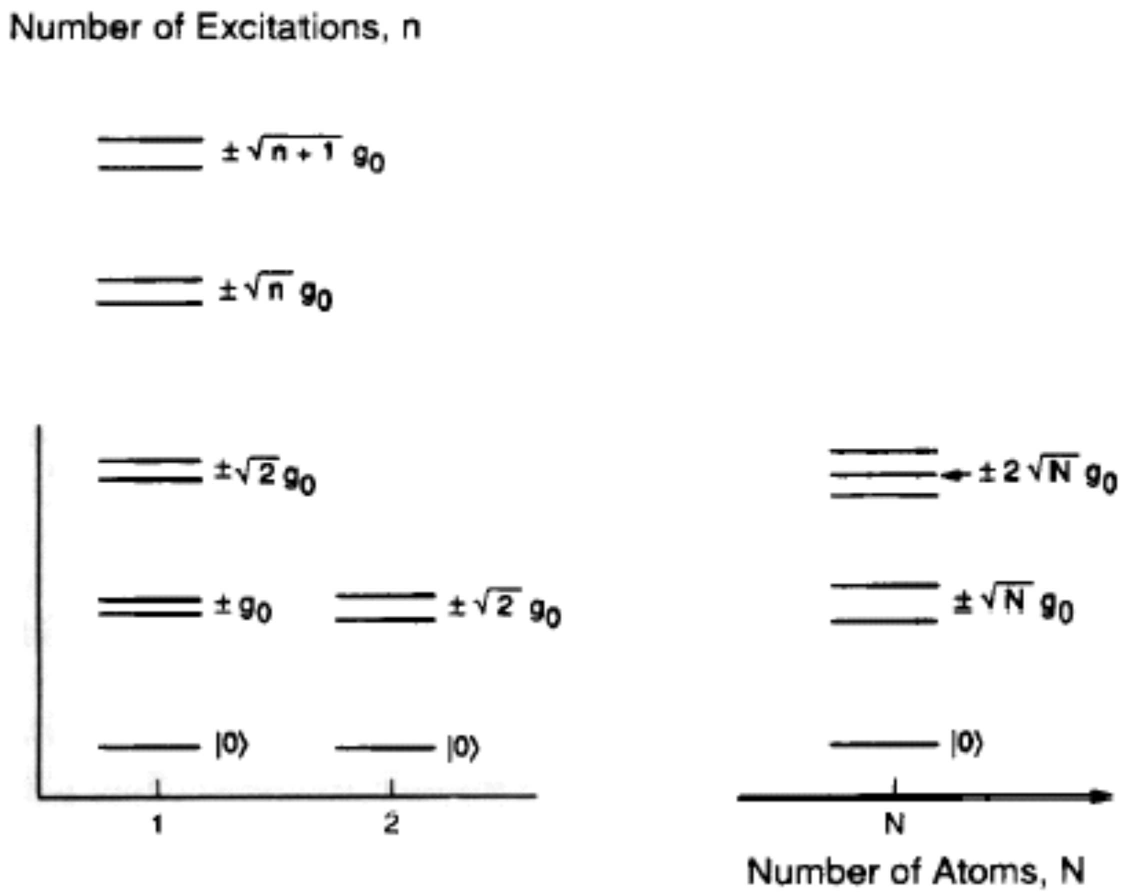
Conditional evolution of the state

$$\left|\Psi_{ss}\right\rangle = \left|0,g\right\rangle + \lambda\left|1,g\right\rangle - \frac{2g}{\gamma}\lambda\left|0,e\right\rangle + \frac{\lambda^2 pq}{\sqrt{2}}\left|2,g\right\rangle - \frac{2g\lambda^2 q}{\gamma}\left|1,e\right\rangle$$

$$\lambda = \left\langle \hat{a} \right\rangle, \quad p = p(g,\kappa,\gamma) \text{ and } q = q(g,\kappa,\gamma)$$

$$\hat{a}\left|\Psi_{ss}\right\rangle \Rightarrow \left|\Psi_{conditioned}\right\rangle = \left|0,g\right\rangle + \lambda pq\left|1,g\right\rangle - \frac{2g\lambda q}{\gamma}\left|0,e\right\rangle$$

Field   atomic polarization



Number of Excitations, n
$\pm\sqrt{n+1}\,g_0$
$\pm\sqrt{n}\,g_0$
$\pm\sqrt{2}\,g_0$
$\pm g_0$
$\pm\sqrt{2}\,g_0$
$|0\rangle$
$|0\rangle$
1
2
$\pm 2\sqrt{N}\,g_0$
$\pm\sqrt{N}\,g_0$
$|0\rangle$
N
Number of Atoms, N

nature physics

LETTERS

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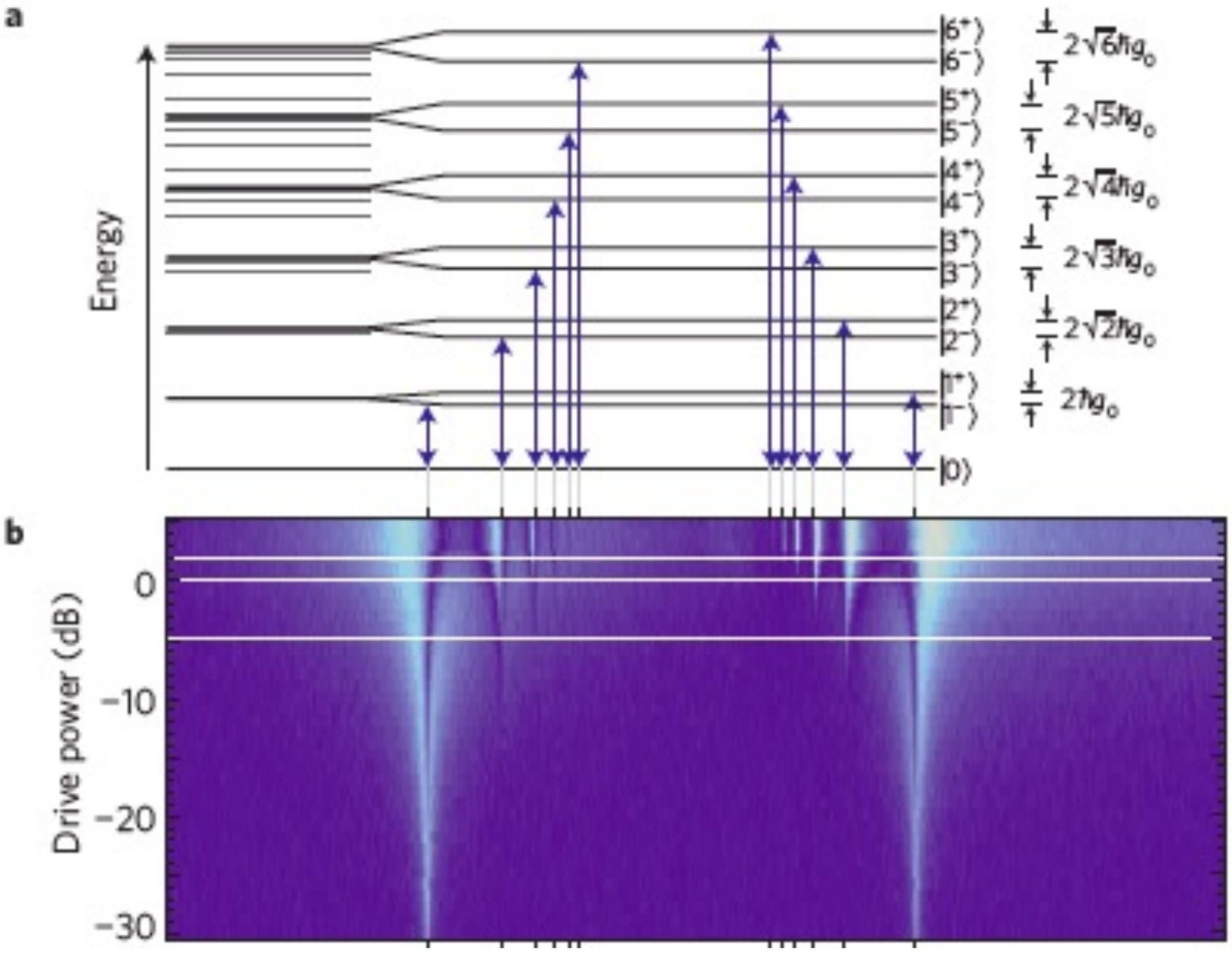
Nonlinear response of the vacuum Rabi resonance

Lev S. Bishop[1], J. M. Chow[1], Jens Koch[1], A. A. Houck[1], M. H. Devoret[1], E. Thuneberg[2], S. M. Girvin[1] and R. J. Schoelkopf[1]*


a
Energy
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|6⁻⟩
|5⁺⟩
|5⁻⟩
|4⁺⟩
|4⁻⟩
|3⁺⟩
|3⁻⟩
|2⁺⟩
|2⁻⟩
|1⁺⟩
|1⁻⟩
|0⟩
$2\sqrt{6}\hbar g_0$
$2\sqrt{5}\hbar g_0$
$2\sqrt{4}\hbar g_0$
$2\sqrt{3}\hbar g_0$
$2\sqrt{2}\hbar g_0$
$2\hbar g_0$
b
Drive power (dB)
0
−10
−20
−30

Thanks